MMAT5010 Linear Analysis (2024-25): Homework 3 Deadline: 15 Feb 2025

## **Important Notice:**

 $\clubsuit$  The answer paper must be submitted before the deadline.

 $\blacklozenge$  The answer paper MUST BE sent to the CU Blackboard.

- 1. Let  $\|\cdot\|$  and  $\|\cdot\|'$  be the norm functions defined on a vector space X. Let D be a subset of X. Show that if  $\|\cdot\| \sim \|\cdot\|'$ , then D is compact with respect to the norm  $\|\cdot\|$  if and only if it is also compact with respect to the norm  $\|\cdot\|'$ .
- 2. Show that the norms  $\|\cdot\|_{\infty}$  and  $\|\cdot\|_2$  are not equivalent on the finite sequence space  $c_{00}$ .
- 3. Let  $(X, \|\cdot\|_X)$  and  $(Y, \|\cdot\|_Y)$ . For each element  $(x, y) \in X \oplus Y$ , put

 $||(x,y)||_{\infty} := \max(||x||_X, ||y||_Y)$  and  $||(x,y)||_1 := ||x||_X + ||y||_Y.$ 

Show that these two norms are equivalent on  $X \oplus Y$ .